



Sensitivity and Uncertainty Analysis

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Definitions

- **Sensitivity Analysis** – examines changes in model outputs in response to changes in values of input parameters to ensure that the model is responding properly
- **Uncertainty Analysis** – evaluates and quantifies the uncertainty in model predictions so that it can be considered when using model predictions for decisionmaking



Sensitivity Analysis - Overview

- Practical check on model behavior
- Quantifies response of model to changes in input parameter values
- Compares response to known physics and characteristics of system
- Examines model response for “reasonableness”
- Not a pass-fail test
- Informs choice of variables for uncertainty analysis
- Performed for HSPF and FCM in Calibration Report – EFDC in Validation Report



Sensitivity Analysis - Approach

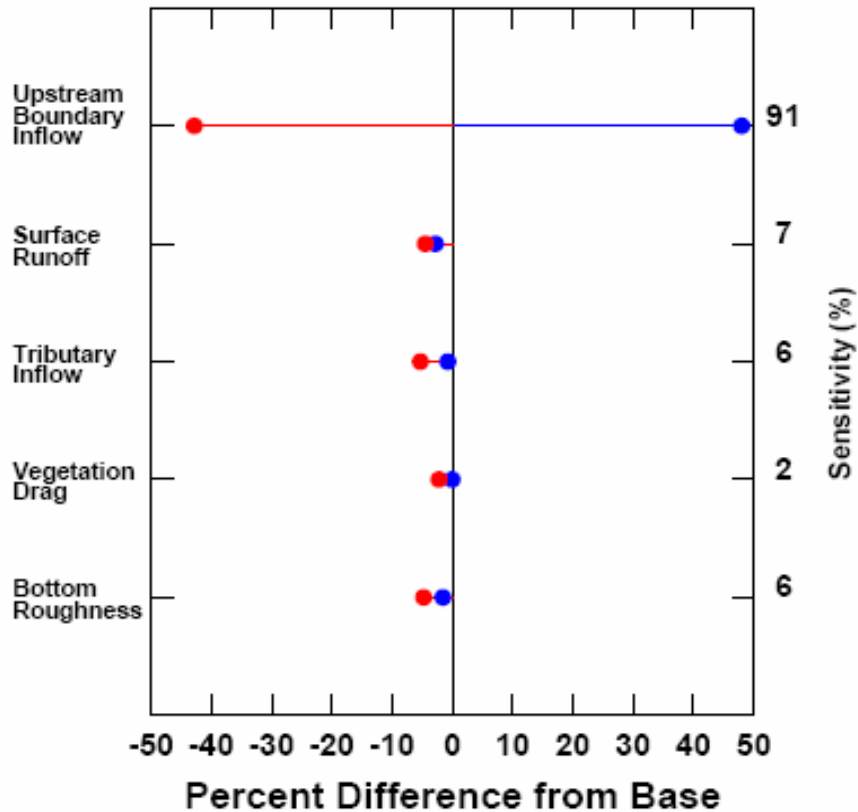
- Select a group of input parameters known or expected to be of primary importance
- Select one or more output variables of interest
- Vary individual input parameter values by $\pm 50\%$ and examine response (direction and magnitude) of output variables
- Present results as “tornado diagrams” and examine for reasonableness



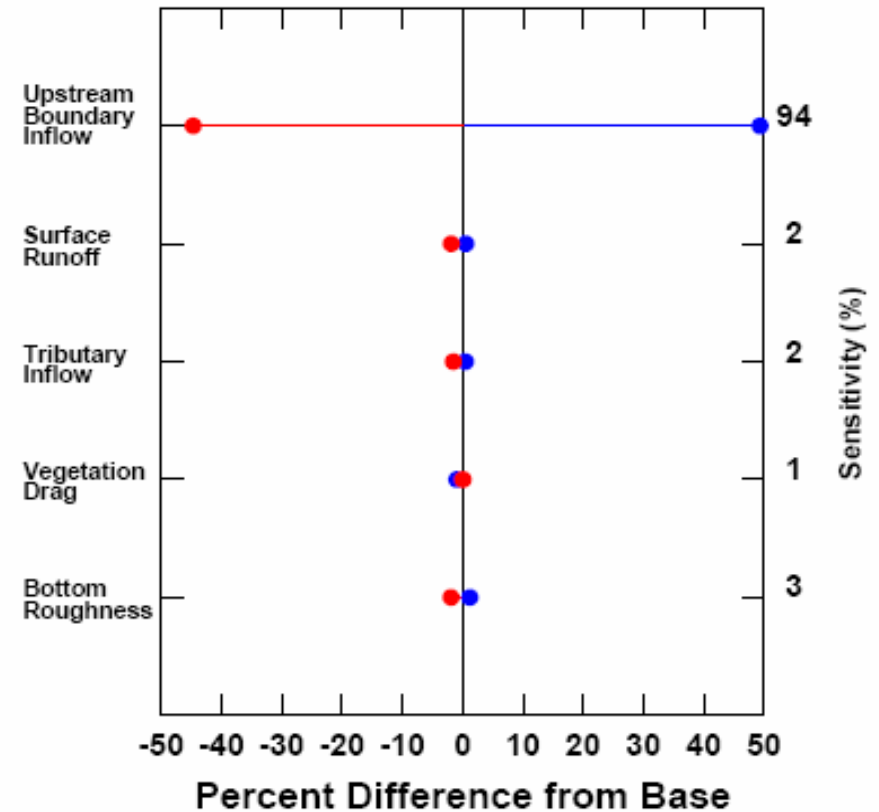
Hydrodynamics at New Lenox Road

NEW LENOX ROAD

Peak Flow



Peak Depth

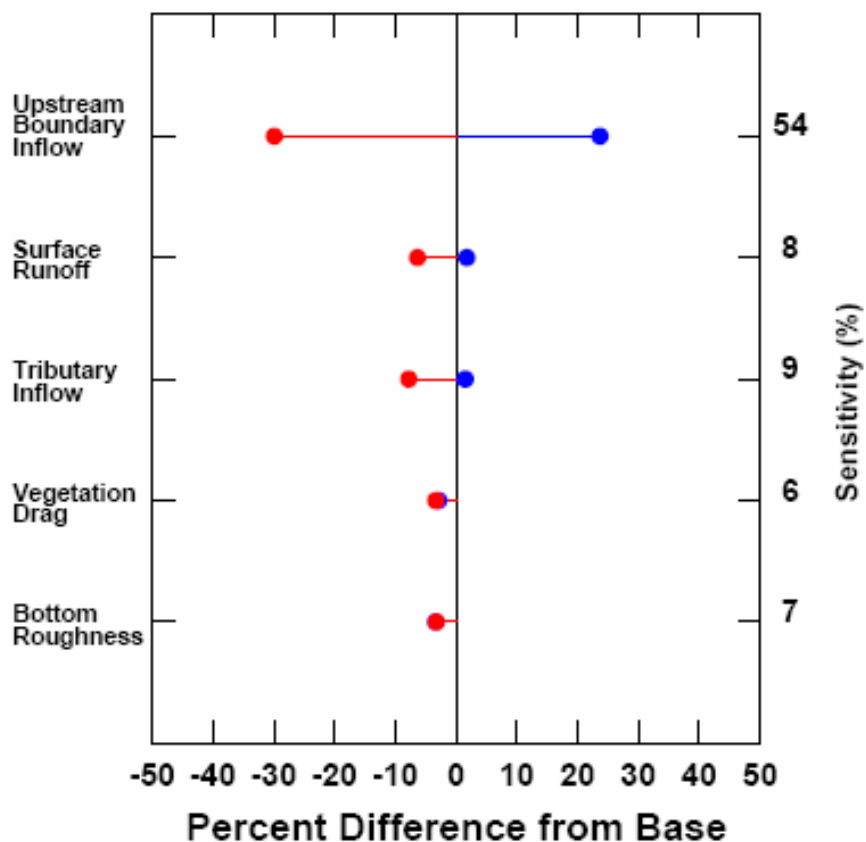




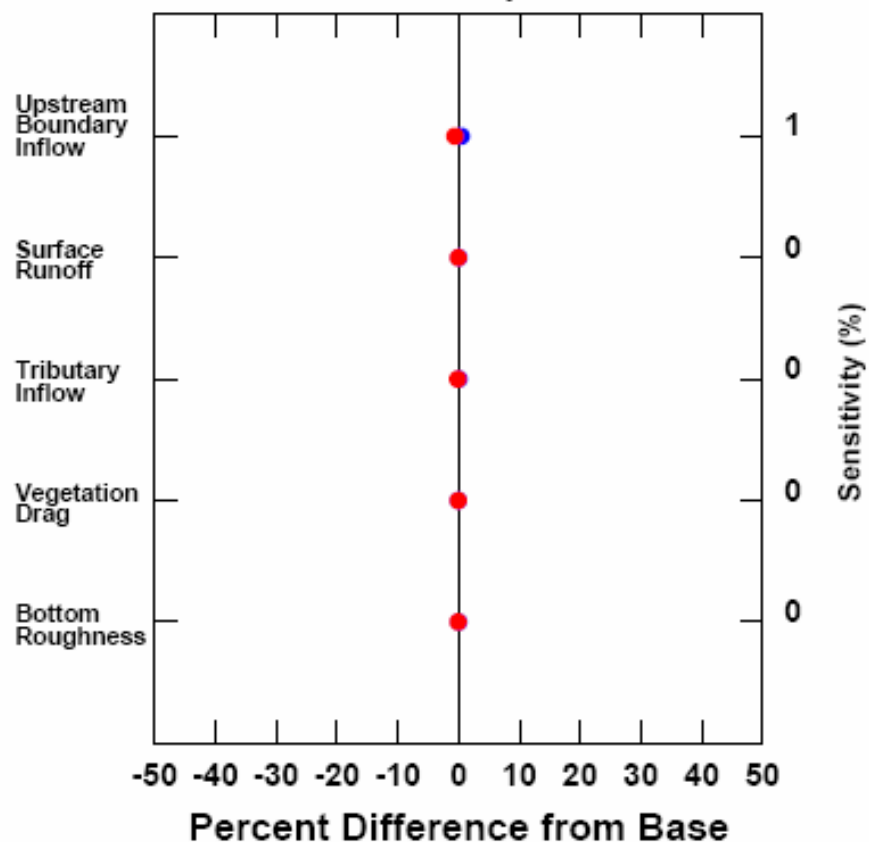
Hydrodynamics at Woods Pond

WOODS POND FOOTBRIDGE

Peak Flow



Peak Depth



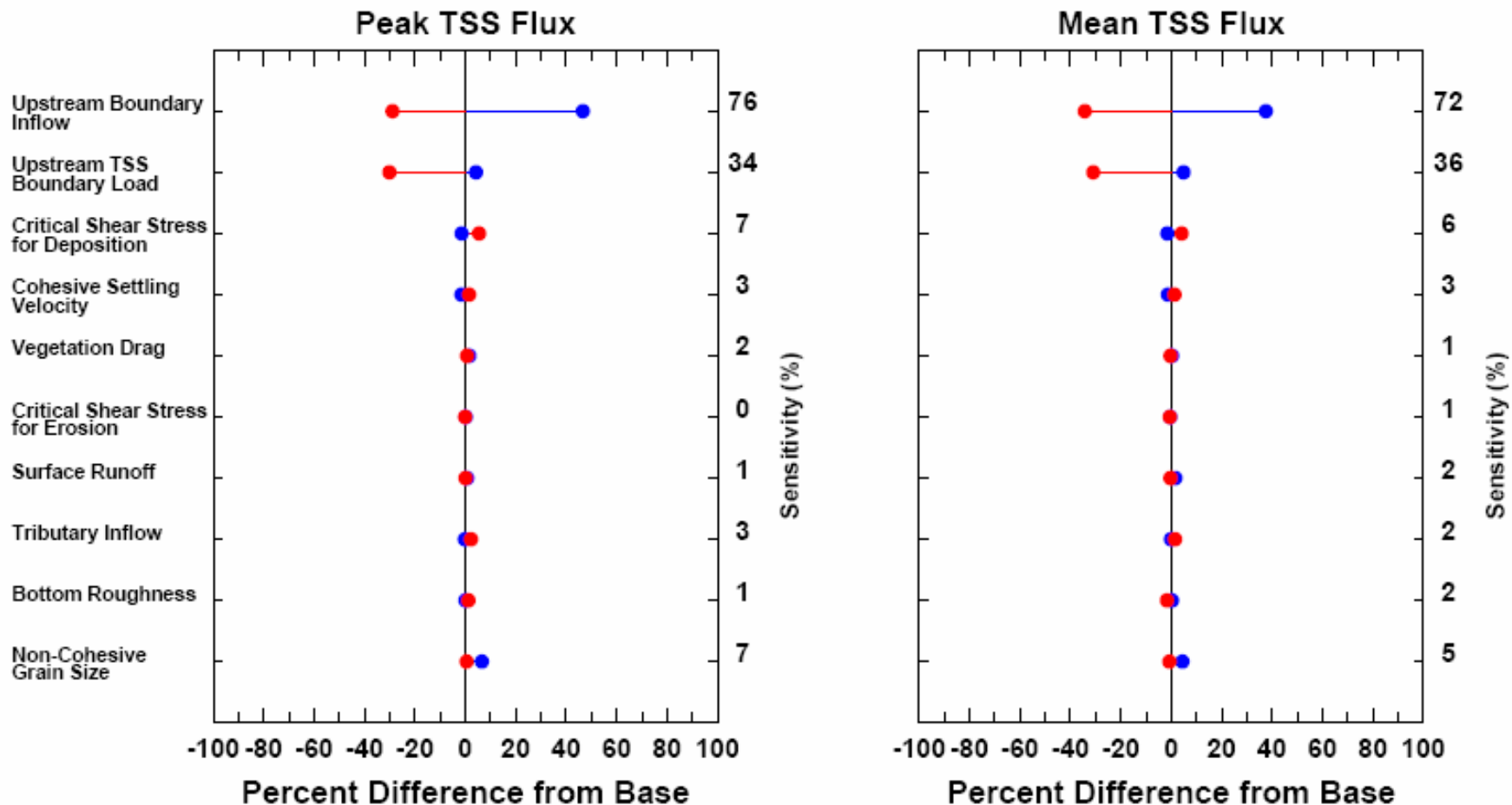
Blue = 50% increase in parameter

Red = 50% decrease in parameter



Solids Transport – Low Flow

NEW LENOX ROAD

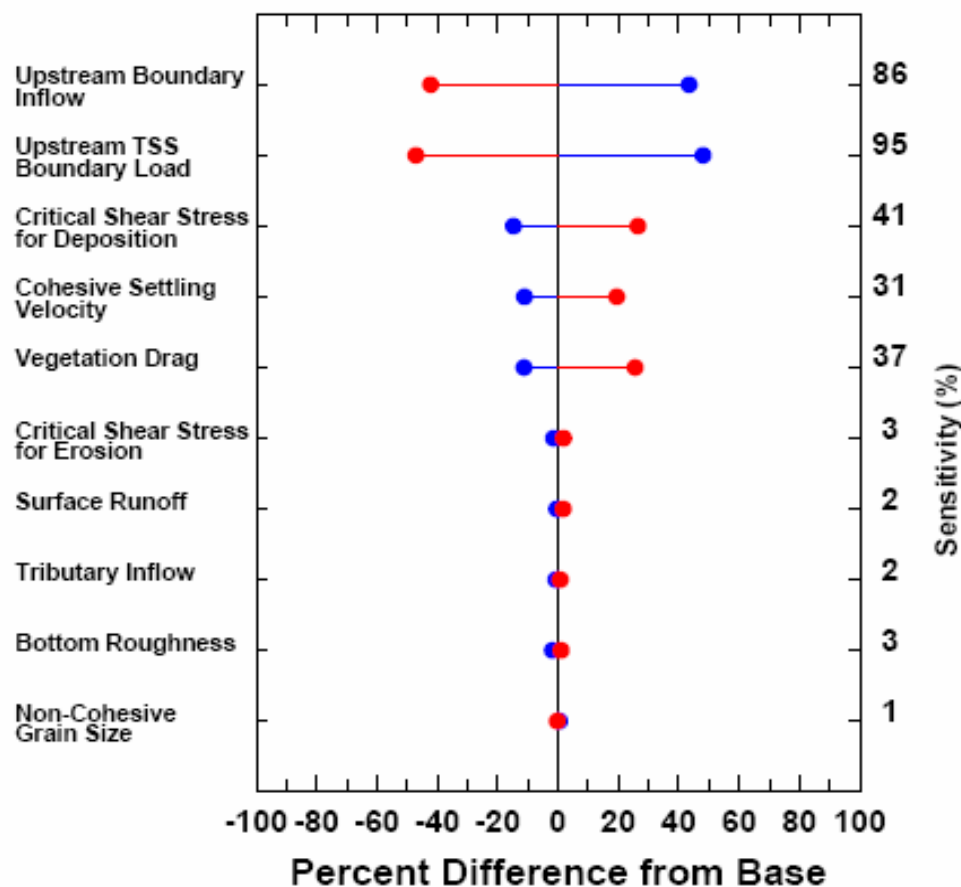




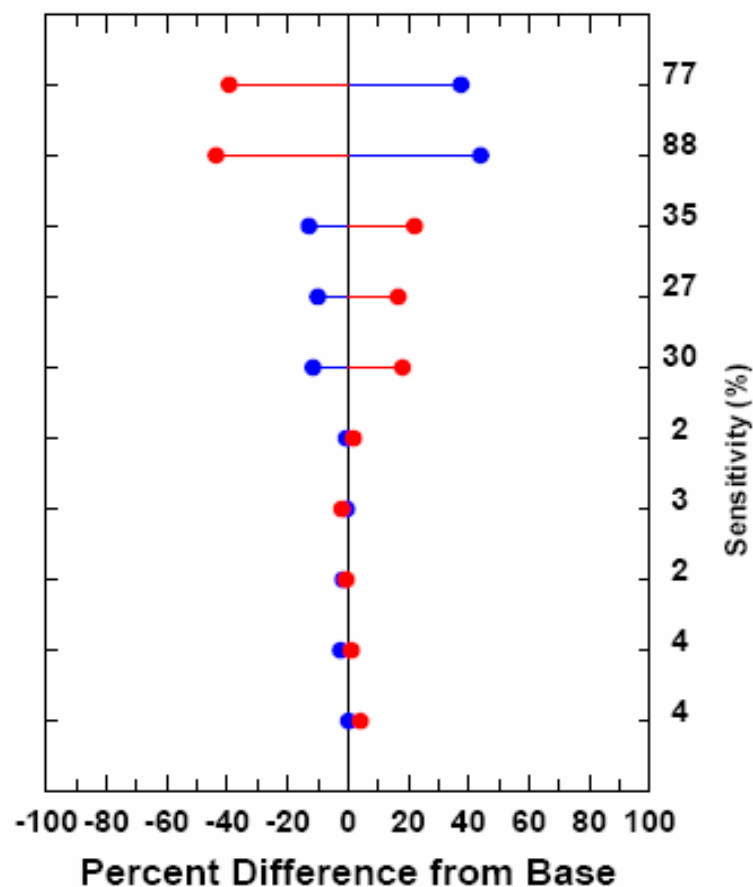
Solids Transport - High Flow

NEW LENOX ROAD

Peak TSS Flux



Mean TSS Flux

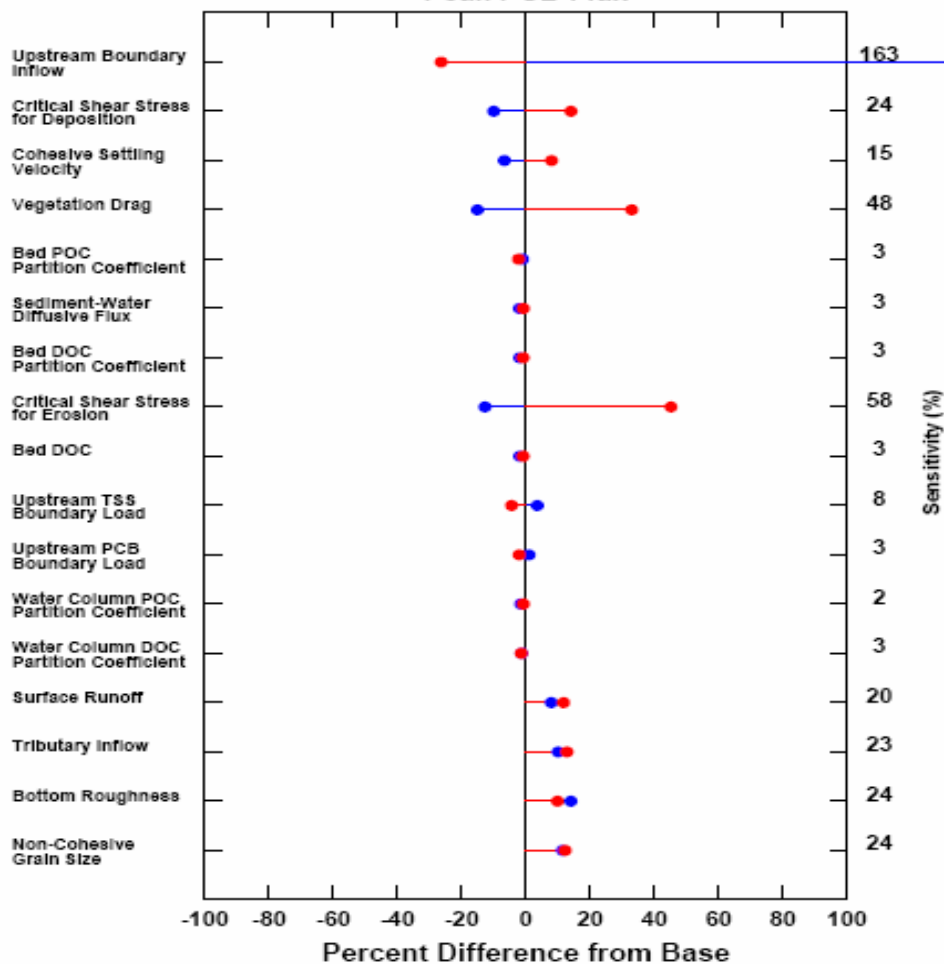




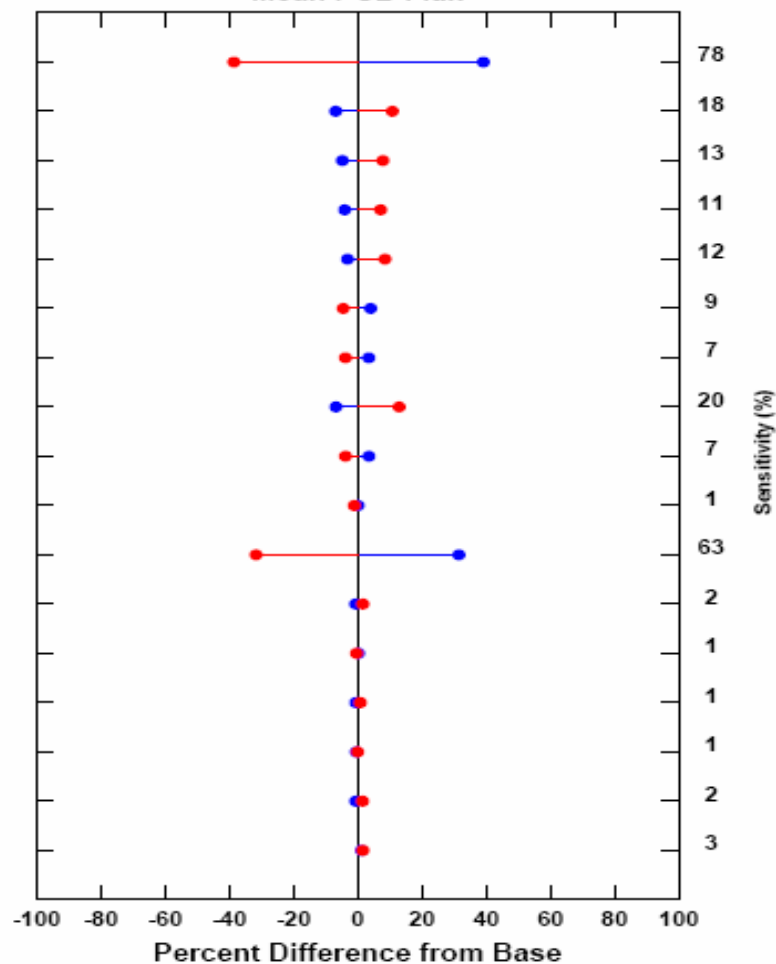
PCB Transport – Entire Period

NEW LENOX ROAD

Peak PCB Flux



Mean PCB Flux



Blue = 50% increase in parameter

Red = 50% decrease in parameter



Uncertainty Analysis - Overview

- Uncertainty is normal (the real world is stochastic)
- Uncertainty analysis quantifies the uncertainty in model predictions
- Informs model users concerning the accuracy of those predictions
- No uniformly accepted procedures for uncertainty analysis of numerical models
- Uncertainty analysis never previously conducted for a model of this size and complexity



Understanding Uncertainty

$<2 = 0\%$

$>12 = 0\%$



Most likely result = 7

$1/6 = 16.6\%$

Result between 6 and 8
 44.3%

Using an understanding
of uncertainty to make
an informed decision

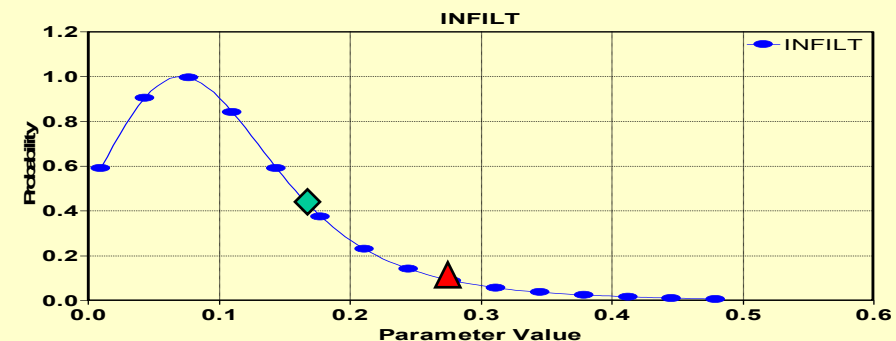
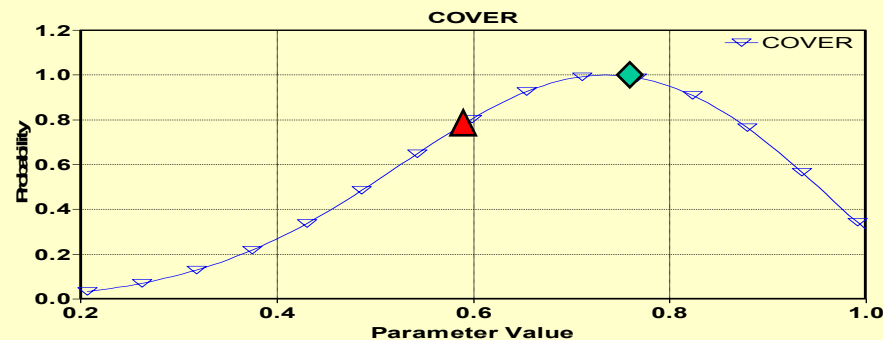
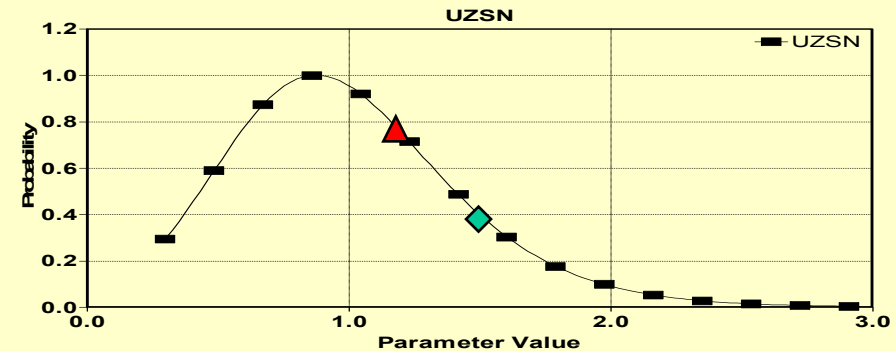
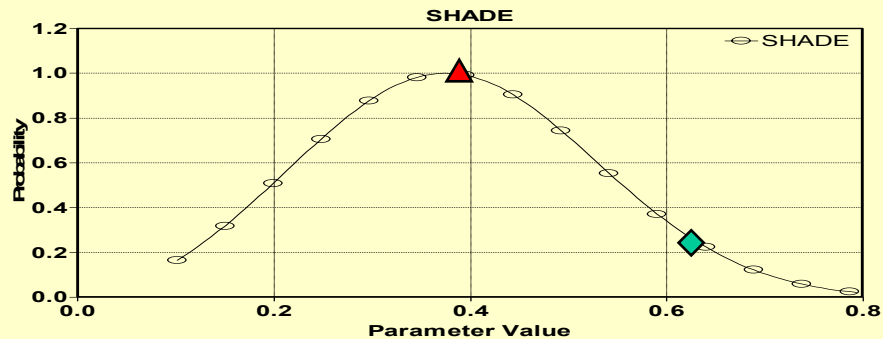
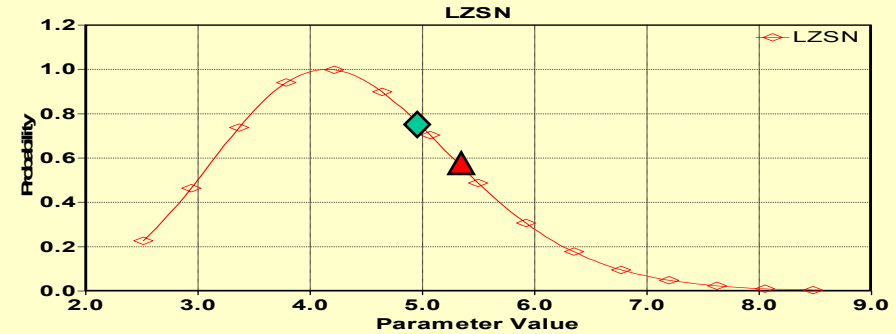
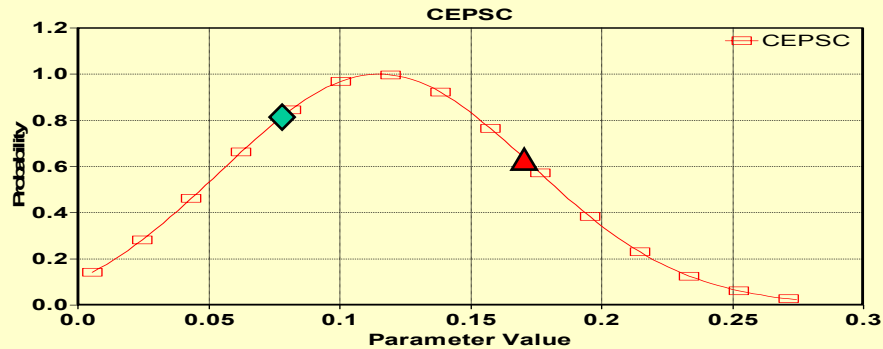


Uncertainty Analysis - Approach

- Monte Carlo Analysis (MCA)
 - Create distributions for input parameters
 - Run model simulation many times, sampling from each distribution
 - Each run produces a value for an output variable of interest
 - Many runs allow creation of an output distribution
 - Appropriate for HSPF and FCM
 - Same approach as used in HHRA and ERA
- Kolmogorov-Smirnov (KS) Analysis
 - Places bounds on a distribution
- Response Surface Model (RSM)
 - Creates a “model of the model” – then analyzed by MCA
- KS and RSM used for EFDC
- Uncertainty for linked parameters propagated through the analysis



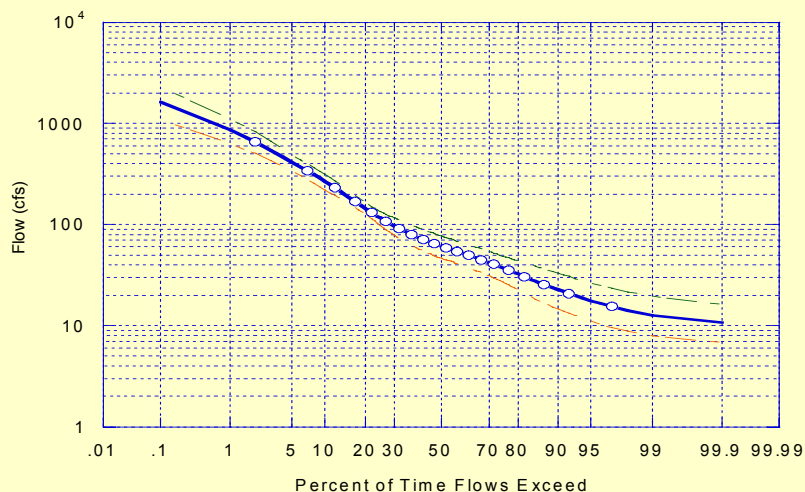
Watershed Model Input Distributions – MCA Example



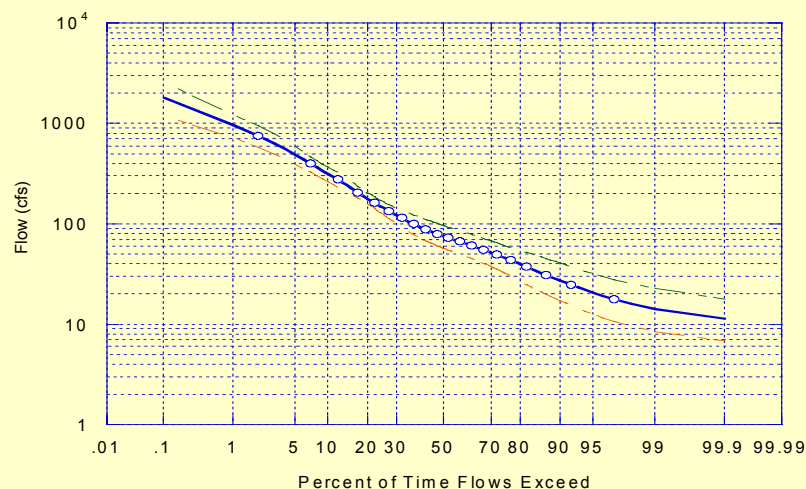


HSPF Uncertainty Results

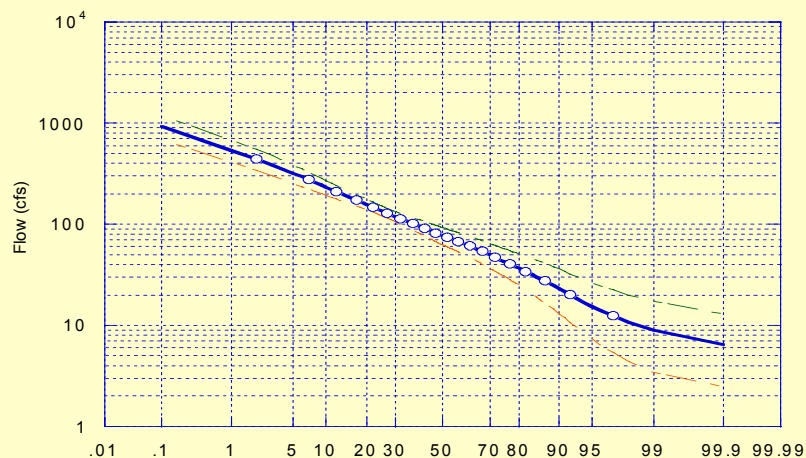
**Coltsville
Flow Duration Curve
(Reach 110)**



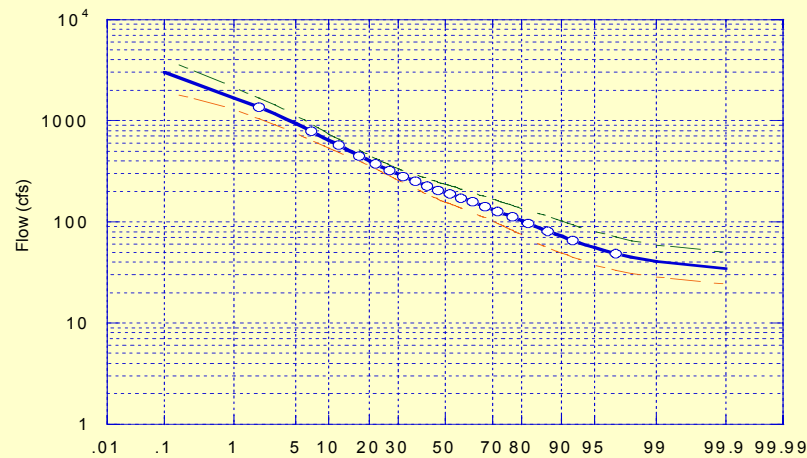
**Pomeroy
Flow Duration Curve
(Reach 400)**



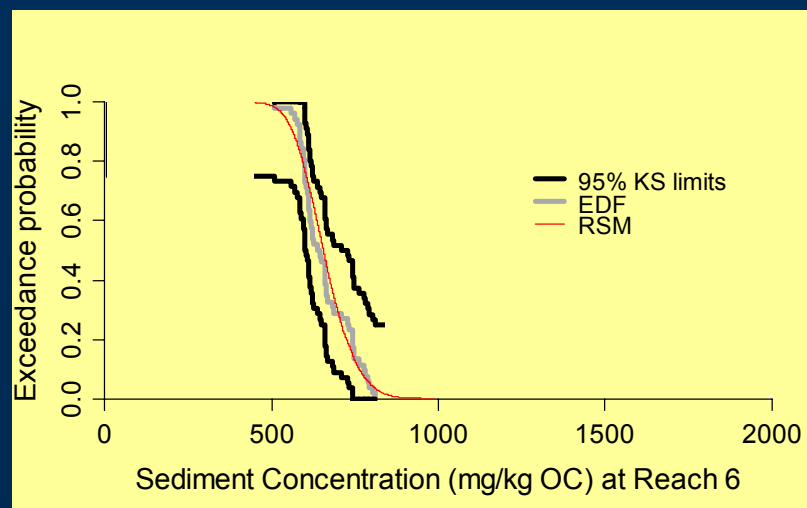
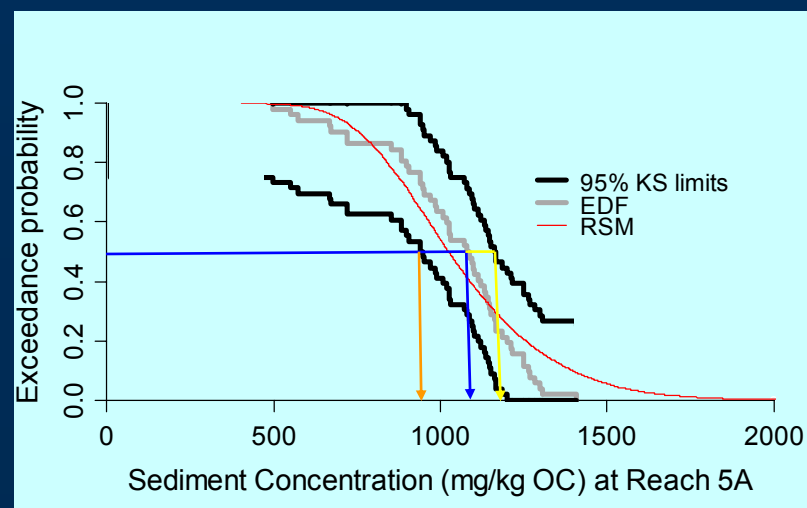
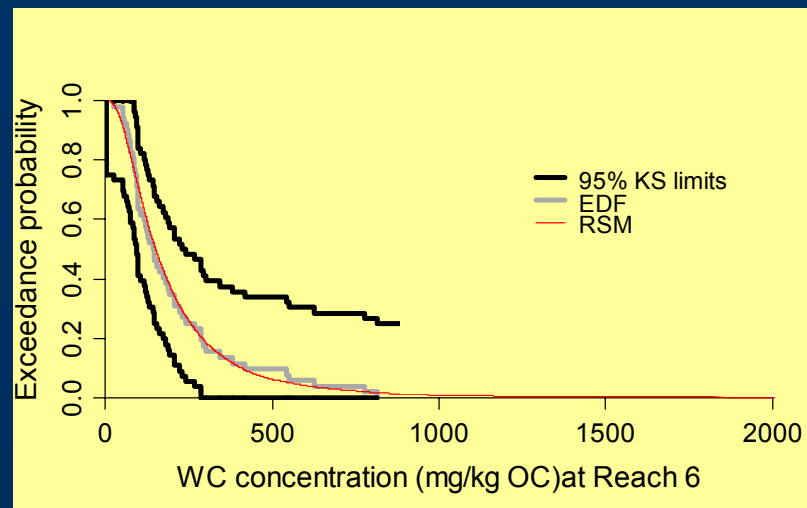
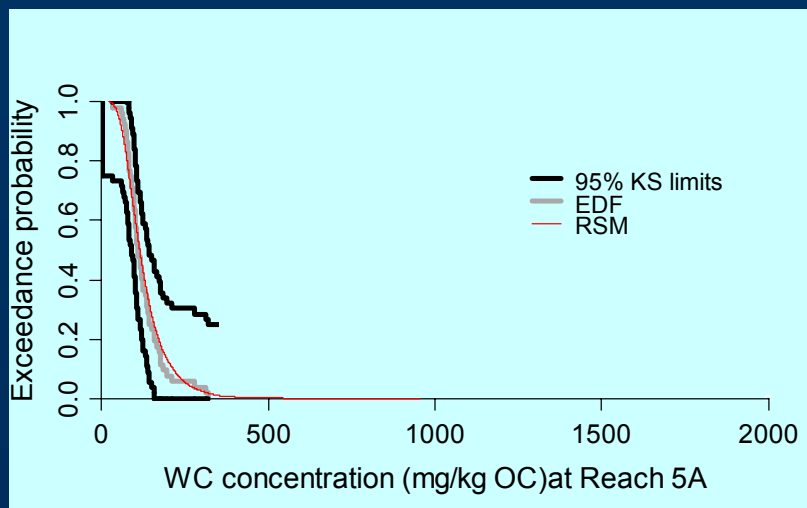
**West Branch
Flow Duration Curve
(Reach 820)**



**New Lenox Rd
Flow Duration Curve
(Reach 540)**



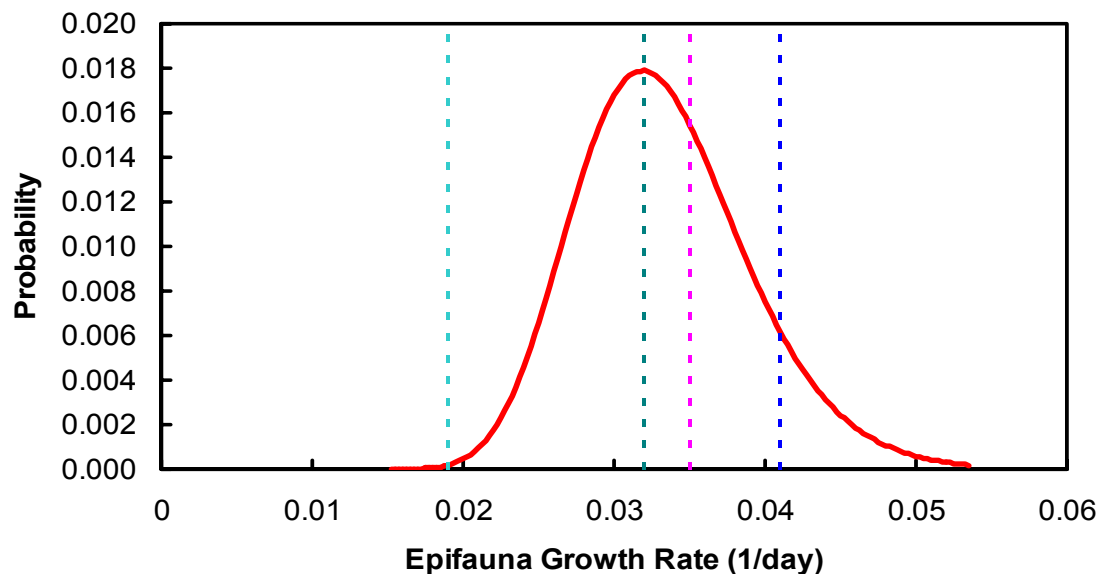
EFDC Uncertainty Results





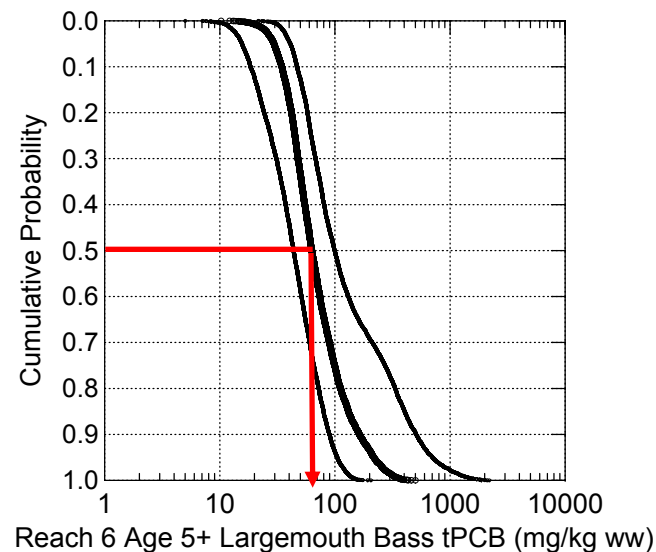
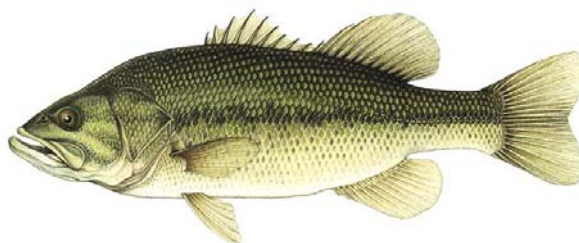
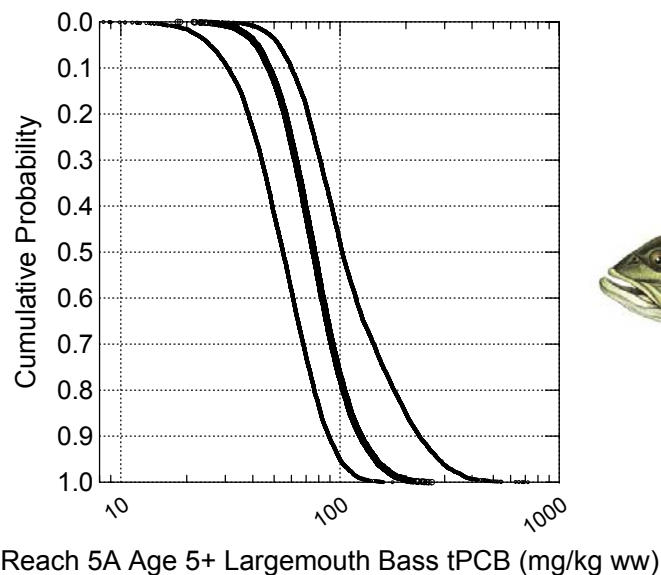
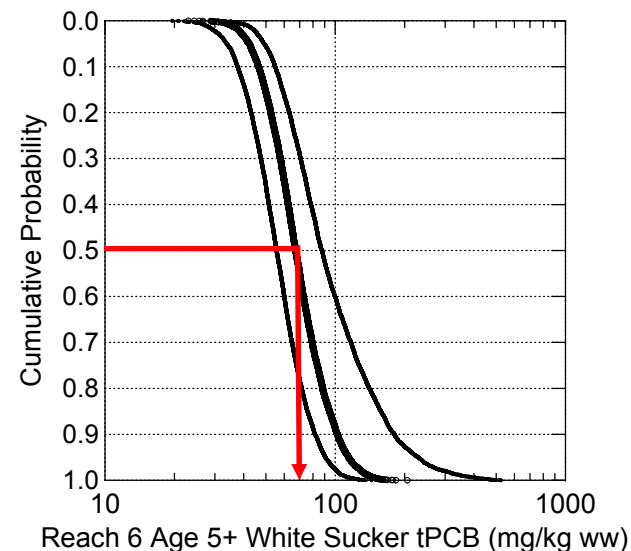
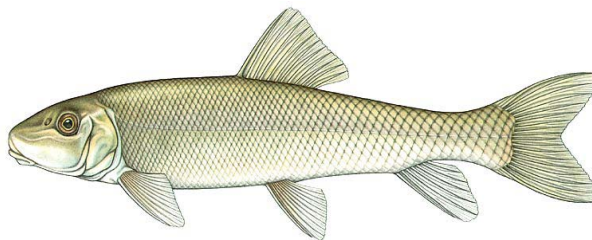
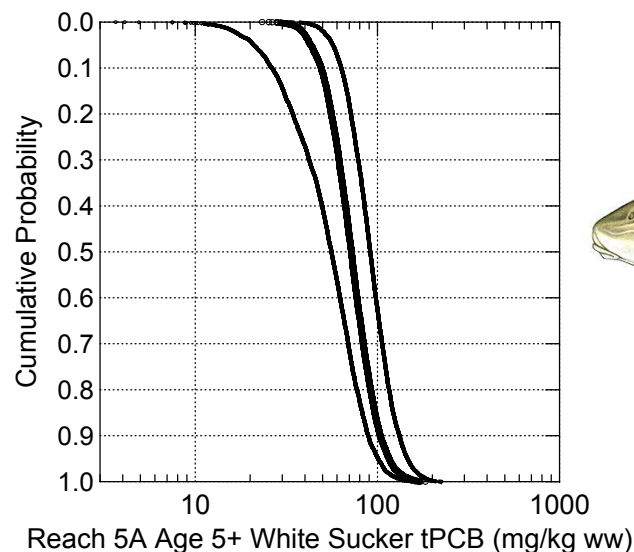
FCM Uncertainty Input Distributions

1. Invertebrate feeding preferences
2. Fish feeding preferences
3. Octanol:water partition coefficient
4. Fish respiration rates
5. Invertebrate assimilation efficiency
6. Invertebrate respiration rate
7. Invertebrate growth rate
8. Biphasic resistance factor for fish
9. Fish assimilation efficiency
10. Energy density of sediment (KJ/g OC)
11. Energy density of POM (KJ/g OC)
12. POM tPCB from EFDC (mg/kg OC)
13. Sediment tPCB from EFDC (mg/kg OC)



- Probability Density Function
- - - Mean of Amphipod Studies (n = 10)
- - - Mean of Caddisfly Studies (n = 6)
- - - Mean of Mayfly Studies (n = 12)
- - - Mean of Miscellaneous Epifauna Studies (n = 5)

FCM Uncertainty Output





Conclusions

- Sensitivity Analysis indicates that all three models (HSPF, EFDC, FCM) respond reasonably based on physics of the Housatonic River system.
- Innovative approach to linked model uncertainty worked well.
- Uncertainty Analysis provides quantitative measures of uncertainty that can be used to inform decision-making.